Final project

Statistical Mechanics – Fall 2010

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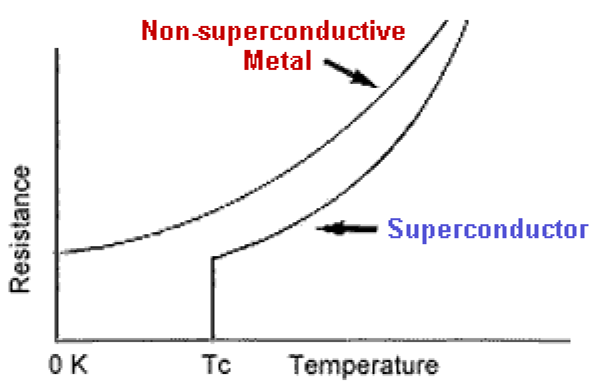
Superconductivity

Introduction

Superconductivity refers to the phenomenon of near-zero electric resistance exhibited by conductors as their temperature is dropped below a certain critical value. Due to lack of dissipation, currents in such conductors can be observed to flow for years after once they are set up. The property is shown by different metals and alloys, each having its own characteristic critical temperature Tc . For example, mercury shows superconductivity below Tc = 4.1K whereas silver never does. A generic variation of resistance with temperature is shown in the graph below. The following pages will describe the basic theory of superconductivity oriented on the formalism of classical electricity and magnetism as well as quantum statistics. Emphasis has been laid on a qualitative description and only the essential Mathematical details have been given.



Conduction models and Quantum mechanics

Classical theories of metallic conduction treated electrons as a gas of particles colliding with lattice imperfections. Charges move together under the influence of an electric field with a certain drift velocity giving rise to what we call current.

$$I = neAv;$$

I = Current, n = no. of electrons, A= cross-sectional area of conductor, v = drift velocity

Quantum mechanics takes into account the wave nature of the electrons and employs the free electron model to account for resistance to electron flow. The model, however, cannot explain superconductivity. Before, describing a holistic theory of superconductivity we need to look at the following aspects of it.

Resistivity

Since the phenomenon of superconductivity involves a drop in the resistance of the material, we need to have a microscopic look at what constitutes resistivity in a solid itself. The main two sources of electron scattering within a solid are:

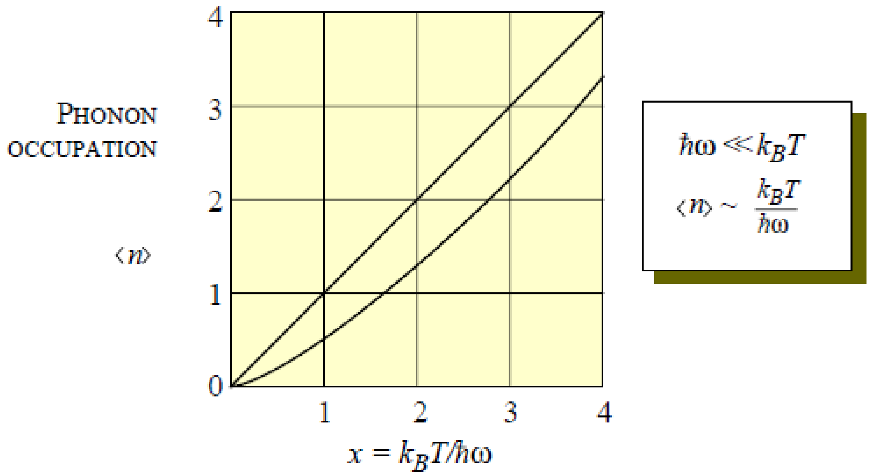
1) Lattice Imperfections. Deviations from a perfect crystal structure induced by impurities for instance can offer resistance to smooth flow of charge.
2) Electron-phonon interactions. In a real crystal atoms are not fixed at rigid sites on a lattice, but are vibrating. In a periodic structure the vibrations have a waveform (just like electronic wave functions) with a spatial and temporal part:

$$u(r,t) = C \exp(ik \bullet r) \exp(-it)$$

$k = 2\pi/h$ represents a “quantum” of vibration energy.

Each vibrational mode constitutes a phonon through the lattice that can interact with an electron and scatter it. The number of phonons propagating through a crystal is a function of temperature, as demonstrated by both the Bose-Einstein and Fermi Dirac statistics

$$\langle n(\omega)\rangle = 1/\exp(h\omega/kT) \pm 1$$



Thus, a correlation between temperature and the number of phonons hints at why resistivity may fall to zero at very low temperatures. There's one more effect that we need to describe before giving solid theoretical explanation of superconductivity.

Meissner effect

Below the critical temperature or when a material makes a transition to the superconducting phase, it expels all magnetic flux from its interior. The Meissner effect may be explained using the 'London equation'. It relates the curl of the current density J to the magnetic field:

$$\vec{\nabla} \times \vec{J} = -\frac{1}{\mu_0 \lambda_L^2}\vec{B}$$

By relating the London equation to Maxwell's equations, it can be shown that the Meissner effect arises from the London equation. One of Maxwell's equations is

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Using the vector calculus identity

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = -\nabla^2 \vec{B}$$

along with the curl of the Maxwell equation above

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \vec{\nabla} \times (\mu_0 \vec{J}) = -\frac{\vec{B}}{\lambda_L^2}$$

and by substitution

$$\nabla^2 B = \frac{B}{\lambda_L^2}$$

By solving the above equation, it can be shown that the magnetic field exponentially decays to zero inside the superconductor. The London penetration depth λ is the length required for the external magnetic field to fall to 1/e of its value.

If the external magnetic field exceeds a certain critical value, the metal ceases to be superconducting. The value of this critical field is also temperature dependent. It is the value at which the energy of the superconducting state exceeds that of the normal state and hence the transition to non-superconductivity. Feynman explains the critical field in the following way. The Gibbs function is defined as G = F + P V where P is the pressure and V is the volume. Here the pressure can be taken to be the energy per unit volume required to push the field out. From classical thermodynamics, the Gibbs function has the property that in a reversible change of phase at constant temperature and pressure, G does not change. So the critical field is characterized by

$$F_{\text{supercond}} + \frac{\mu H_{cr}^2}{8\pi} V = G_{\text{supercond}} = G_{\text{normal}} = F_{\text{normal}}.$$

At the critical temperature, Hcr= 0, and as the temperature decreases Fnorm increases, so that Hcr increases.

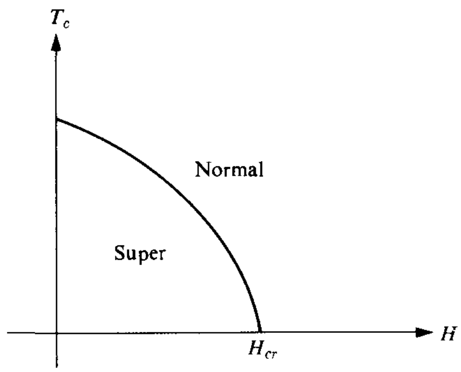


Fig. 10.2 Variation of critical field with temperature.

BCS theory

The independent particle model to explain conductivity in metals ignored lattice vibrations. However, we know that the critical temperature is in fact related to the isotopic mass 'M' of the element used as the superconductor, from which we know that the lattice vibrations have a role to play.

$$M^{1/2} T_c = \text{constant}$$

One self-consistent theory that takes into account these vibrations and relies on electron-phonon interactions to explain superconductivity was proposed by Bardeen, Cooper and Schrieffer in 1957, now called the BCS theory. According to this theory, under certain conditions the attraction between two electrons due to a succession of phonon exchanges can exceed their mutual Coulomb repulsion, making them weakly bound together to form Cooper –pairs that are responsible for superconductivity.

Formation of Cooper pairs

The crux of phonon-electron interaction can be described as follows. In a metallic lattice, the positively charged ions form a region of increased charge density carrying momentum and propagate as a travelling wave through the structure. This effect is brought about by a passing by electron and its Coulomb attraction, and that is why is described as the emission of a phonon by the particular electron. This phonon can be absorbed by a second electron when it experiences Coulomb attraction from the moving region of positive charge density and absorbs the momentum. On the whole, the two electrons can be seen to have exchanged some momentum and participated in an attractive interaction with each other. The result is a Cooper pair. We can now define certain conditions that are necessary for the formation of a Cooper pair:

1) Low temperature in order to inhibit random thermal phonons.
2) Strong electron-phonon interaction.
3) A large number of electrons lying just below the Fermi energy. At T=0, electrons below Fermi energy promote themselves to vacant states just above Ef to form Cooper pairs.
4) Anti-parallel spins to allow symmetric eigenfunctions and consequently the proximity of electrons in space.
5) The two electrons of a pair should have equal and opposite momenta.

Order parameter

When an electric field is applied across the superconductor, electron pairs move under its influence maintaining order – their centers of mass have the same momentum in order not to be disturbed by lattice imperfections. Below the critical temperature, the superconducting ground-state is one large-scale quantum state in which the motions of electrons are highly correlated. A Cooper pair can therefore be defined to share a common wave function ψ(r) or the order parameter. ψ(r) can be thought of as a measure of the order in the superconducting state below TC. It has the properties that ψ(r) goes to 0 as T approaches the critical temperature, and its modulus squared approaches the Cooper pair density.

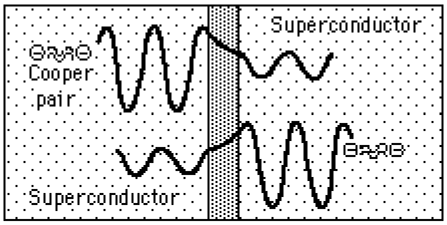
Binding energy

The binding energy of a Cooper pair at absolute zero is about 3kTc. As the temperature rises, the binding energy is reduced and becomes zero at the critical temperature. Any source needs to provide a minimum energy Eg – the binding energy of the Cooper pair for the pair to break.

Observation of superconductivity

“Two superconductors separated by a thin insulating layer can experience tunneling of Cooper pairs of electrons through the junction. The Cooper pairs on each side of the junction can be represented by a wave-function similar to a free particle wave function. In the DC Josephson effect, a current proportional to the phase difference of the wave-functions can flow in the junction in the absence of a voltage. In the AC Josephson effect, a Josephson junction will oscillate with a characteristic frequency which is proportional to the voltage across the junction.

Since frequencies can be measured with great accuracy, a Josephson junction device has become the standard measure of voltage. “



Applications

Superconductivity is an immensely useful phenomenon because of an absence of power dissipation. Superconducting electromagnets are used in electric motors and generators. Because superconductors are diamagnetic, they can be used to shield out unwanted flux. This has been put to use in shaping the magnetic lens of an electron microscope to eliminate stray field lines and to greatly improve the practical resolving power of the instrument. Another great use of superconductivity is n Superconducting Quantum Interference Device or SQUID’s that can be used to detect minute changes in magnetic flux and detect extremely small fluctuations in voltage and current.

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